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What is Resource Theory ?

- Free States
- Free Operations
- Examples
  - RT of entanglement
    (Separable states, LOCC / SEP maps)
  - RT of coherence
    (incoherent states, I.O.)
  - RT of purity
    (completely mixed state, noisy operations)
  - RT of reference frames/ Symmetry

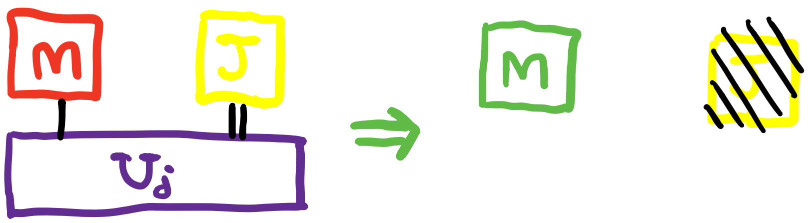
    $$U_g \sigma U_g^\dagger = \sigma \quad \forall g \in G$$

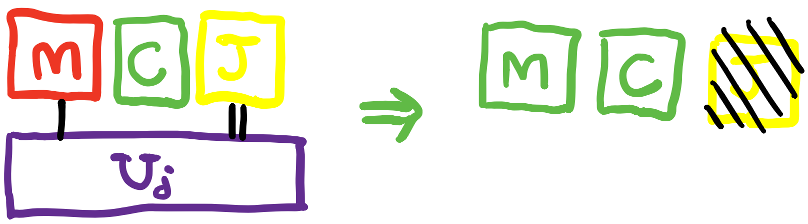
  - RT of q. thermodynamics
    ($\{e^{-\beta H}\}$, $[U, H] = 0$)

General RT

- $\mathcal{F}$ is convex
- $\mathcal{F}$ is closed under tensor Product.
- $\mathcal{F}$ is closed under Partial trace.

"How Resourceful" a state is ?





- $(\varepsilon, \log|J|)$-catalyst transformation

$\exists\ \sigma_M \in \mathcal{F}$ s.t.

$$\mathrm{dist}\left(\omega_{MC}, \sigma_M \otimes \mu_C\right) \le \varepsilon$$

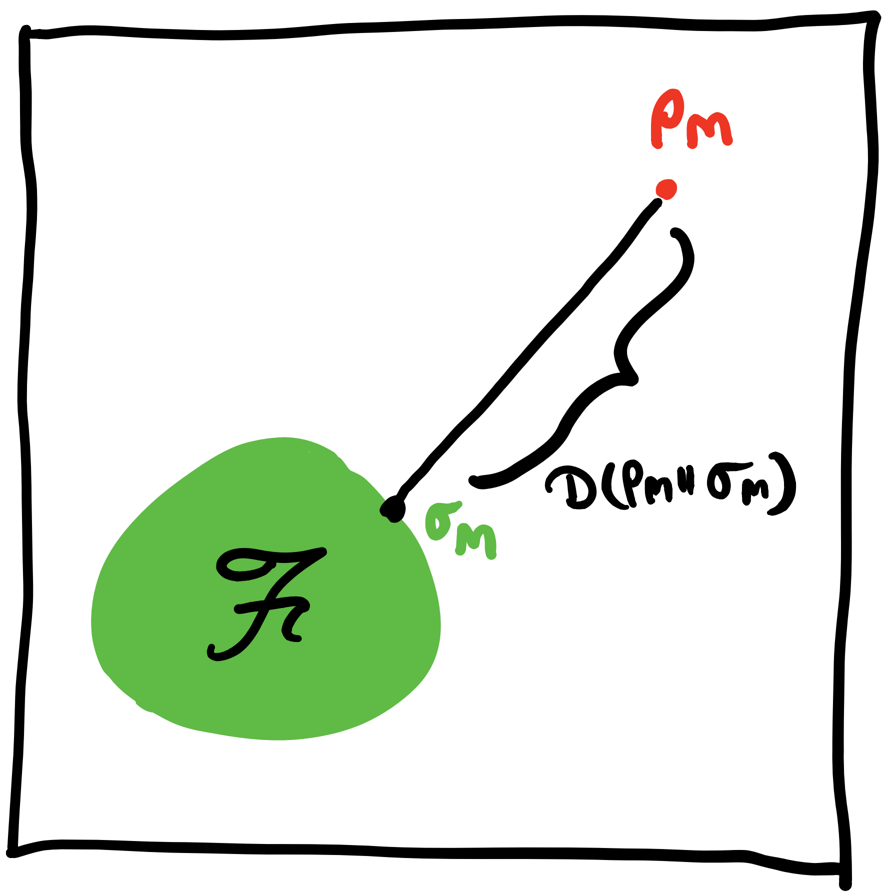
where

$$\omega_{MC} = \sum_{j=1}^{|J|} U_j \left(\rho_M \otimes \mu_C\right) U_j^{\dagger}$$

- $$R(\rho_M) \equiv \lim_{\varepsilon \to 0} \liminf_{n \to \infty} \left\{ \frac{1}{n} \log|J| : (\varepsilon, \log|J|)\text{-c.t. of } \rho_M^{\otimes n} \to \mathcal{F} \text{ exists} \right\}$$

Relative Entropy of Resource

$$E(\rho_M) = \inf_{\sigma_M \in \mathcal{F}} D(\rho_M \| \sigma_M)$$



Main Results

$$R(\rho_M) = E^{\infty}(\rho_M)$$
$$:= \lim_{n\to\infty} \frac{1}{n} E(\rho_M^{\otimes n})$$

Proposition: Fix $\varepsilon, \delta > 0$ and ρ_M.

- $\exists$ $(\varepsilon+\delta, \log k)$-catalyst transf. of ρ_M to $\mathcal{F}$, where
$$\log k = \min_{\sigma_M \in \mathcal{F}} D^{\varepsilon}_{max}(\rho_M \| \sigma_M) + 2\log\frac{1}{\delta}$$
- $\forall$ $(\varepsilon, \log|J|)$-transformation of ρ_M to $\mathcal{F}$,
$$\log|J| \geq \min_{\sigma_M \in \mathcal{F}} D^{\varepsilon}_{max}(\rho_M \| \sigma_M)$$

Tools

- $D_{max}(\rho_M \| \sigma_M) := \inf \{ \lambda \in \mathbb{R} : 2^{\lambda} \sigma_M \geq \rho_M \}$

  $D^{\varepsilon}_{max}(\rho_M \| \sigma_M)$

  $$:= \sup_{\rho'_M \in B^{\varepsilon}(\rho_M)} D_{max}(\rho'_M \| \sigma_M)$$

- Convex Splitting lemma

  Given ρ_M, σ_M, $k = D^{\varepsilon}_{max}(\rho_M \| \sigma_M)$,

  $$\tau_M^{(n)} = \frac{1}{n} \sum_{j=1}^{n} \rho_{M_j} \otimes \sigma_M^{\otimes [n \setminus j]}$$

  then

  $$\mathrm{dist}(\tau_M^{(n)}, \sigma_M^{\otimes n}) \leq \varepsilon + \sqrt{\frac{2^k}{n}}$$